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5 The Exchange of Voting Positions: An Object-Oriented Model of Policy Networks

In chapter 4, Bueno de Mesquita presented his expected utility theory of forecasting political decisions. In his theory, each actor is assumed to be an expected utility maximizer — that is, each actor evaluates different strategies and pursues the one that he or she believes gives the highest expected utility. His theory allows actors to evaluate strategies solely on one issue at a time. Actors are not given the opportunity to maximize expected utility by connecting their voting positions on one issue to their respective positions on other issues. In this chapter it will be demonstrated that, under certain conditions, two actors can gain expected utility simultaneously by exchanging voting positions on two decisions or issues. Subsequently, a model is presented to predict the exchange rates of such potential trades, to compute the expected utility gains for the actors, and to simulate the realization of exchanges and their effects on the outcomes of decisions in a system of N actors and M decisions.

Our exchange or logrolling model is based on many of the same fundamental assumptions as the Bueno de Mesquita model. In particular, we assume single-peaked utility functions and the unidimensionality of each issue. We focus, however, on the multidimensional linkage across issues rather than on compromises within each separate issue. As our exchange model opens new strategies for actors over several decisions, it can be seen as an extension of the Bueno de

We thank Siegwart Lindenberg, Gwen Moore, Chris Snijders, Jeroen Weesie, Allen Whitt, and Evelien Zeggelink for their extensive comments on earlier drafts. The computer implementation of the model was supported by IBM under a study contract between IBM and the Interuniversity Center for Sociological Theory and Methodology of the University of Groningen, ICS.

Mesquita model. Our model deviates considerably from earlier exchange models of collective decision-making processes that are conceptualized in terms of exchange of control.[1]

Previous Exchange Models

In the early 1970s, Coleman (1972, 1973) presented his well-known social exchange model. Coleman applies Walras's model of economic exchange on a perfectly competitive market to more general exchange processes in decision making by assuming that actor preferences are consistent with a Cobb-Douglas utility function (see Coleman 1990, pp. 674–75). His model is built on a simple conceptual framework consisting of actors and events. Actors vary in their control over and interest in events. Coleman shows how changes at the collective level stem from the exchange between actors of control over one event for control over other events. Exchanges are motivated by differences in the distribution of control over events among the actors compared to their interest in the events. Actors are hypothesized to exchange control over events in which they are less interested for control over events in which they are more interested.

The resources of each actor are defined as his or her amount of control over valued events, whereas the value of an event is defined in terms of the interests of resourceful actors in the event. If the distribution of control over actors and their interests in events are known, the values of the events, the resources of the actors, and the distribution of control at equilibrium can be computed.

Coleman's exchange model is very simple for divisible and private events without market restrictions. Control represents the proportion of that event in the actor's possession. A recent study by Coleman (1990) shows that the model can be fruitfully used for and adapted to a wide variety of social phenomena. One of the most important extensions of his exchange model is its application to indivisible goods in collective decision making. Even in cases of constitutionally fixed sets of rights and multistage procedures of decision making (as in the American Constitution), Coleman states that the system can be modeled in terms of transactions of control among actors:

> The fact that there is generally a stream of collective actions rather than an isolated action means that individuals might be given rights which extend over a class of those actions and allowed to allocate them as they see fit within that

1. The development and application of the model were facilitated by the availability of a modern computer technology, object-oriented modeling, where actors are represented by objects. Like actors in the physical world, they have an internal structure that enables them to reason and to communicate with other objects.

> class. . . . [T]his would constitute a means by which the constraint of indivisibility of actions would effectively vanish: One person would use his resources to gain control of those collective actions that most interested him, another would gain control of those collective actions that most interested him, and so on. (Coleman 1990, p. 373)

The extension of Coleman's model to indivisible goods, however, is neither simple nor straightforward. First, it requires redefinitions of control and interest. Control can no longer be defined as the proportion of an event in the actor's possession if events are indivisible. Instead, control is now defined as the actor's ability to effect an event outcome consistent with his or her preference. In a similar way, interest is linked to the actor's preferences by defining it as the extent to which the well-being of an actor varies with the outcome of that particular event (Podolny 1990, p. 361). Second, Weesie (1987) and Coleman (1990, pp. 822–25) show that interest does not operate in the same way when the interests of actors in an indivisible event are complementary or opposed. This is the first point for which it becomes clear that the extension of the model is not straightforward. In the extension, actors exchange a good (control), the ideal of which lies for all actors in the same direction, characterized by a nondecreasing preference function (the more control, the better). In collective decision making, however, we are dealing with the possibility of opposed preferences and consequently different effects for exchanges among actors depending on whether they have interests that are opposed. Moreover, we are often dealing with single-peaked preferences. For example, when the event concerns a decision on the height of a new public building, actors will have different preferences on the ideal height, and they will oppose a building that is either too high or too low. Therefore, we develop a new model in which opposed and single-peaked preferences are explicitly dealt with. The exchange of control over events remains the heart of all of the adaptations of the original Coleman model to collective decision making. Again, this focal concept is not straightforward in social systems where rights to control are constitutionally fixed and decision making is a multistage procedure.[2]

2. That adjustment of a wrong model might have serious consequences can be seen in the study of Podolny (1990). He — like Marsden (1983) — is primarily interested in the question of which exchange relations among actors are actually established in a collective decision-making situation. He investigates whether such exchanges primarily take place among actors with like interests or opposed interests. In his experiments, he rewards the simple collection of votes by the participants without differentiating between pro and con votes and concludes that exchange relations are more likely to occur among actors with like interests. One may wonder, however, whether any significance should be given to the collection of like votes and, by implication, why the total number

The above observations lead us to the position that Coleman's exchange model is not appropriate for decision making on collective goods. In our opinion, an exchange model is required that takes seriously the following elements of collective decision making and, by implication, uses these as cornerstones in the model:

1. Control is fixed and often embedded in a multistage decision-making procedure. The control of actor i on the outcome of decision a in such procedures is denoted by his or her *voting power* (v_{ia}).
2. Actors differ in their preferences for outcomes over collective decision-making issues. At the highest level of abstraction, actors are assumed to have monotonically increasing utility functions related to universal goals — like physical well-being and social approval — but they have different instrumental preferences for the means that lead to the ultimate goals (Lindenberg 1990, p. 741). In this perspective, outcomes of collective decision making can be perceived as instrumental goals: whereas one outcome can produce social well-being or social approval for one set of people, another outcome can be better for others. In other words, each actor orders outcomes in terms of the contribution the outcome makes to the actor's universal goals. These outcomes are not necessarily dichotomous, but they may well consist of a certain amount of an outcome (e.g., the size of a budget, the height of a new building, or, in the example below, car emission rates). The most preferred position of actor i on decision a is called his or her *policy position* (x^*_{ia}).
3. Actors differ in their interests in decisions. The interest of actor i in decision a is denoted by the *salience* of decision a for the actor i (s_{ia}).
4. Actors ultimately are willing to vote for less preferred policy positions on less salient issues in exchange for a vote for their policy position by other actors on more salient decisions. The final stance of actor i on decision a is called his or her *voting position* (x_{ia}).[3]

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of collected votes has any relevance. What matters in collective decision making is the number of votes that have been reversed so as to favor an actor's own preference on the basis of an exchange.

3. The reader should be aware that our notation differs from Coleman's. He used the symbol x to denote the interest of an actor and the symbol c to denote the control of an actor. The reason for our deviant notation is that the symbol c is used to denote dyadic control relations between actors in the Stokman–Van den Bos model, as is commonly done in social network research, whereas voting power is a relation between an actor and an event. In the Coleman models with opposed interests, the symbol y is

In other words, in collective decision making, outcomes of decisions are instrumental goals for actors, and actors try to maximize their utilities by searching for outcomes that are as close to their policy position as possible. If actors have different saliences and policy positions on decisions, they can produce more utility by exchanging voting positions. This makes sense only when actors have voting power on decisions in which they are willing to support a position other than their own policy preference. This implies that in collective decision making actors do not exchange control, but rather exchange voting positions.

The exchange — or logrolling — model is based on pairwise exchanges between two actors on two decisions. In a larger system of N actors and M decisions, each actor investigates his or her potential exchanges with all other actors on any pair of decisions. Subsequently, which potential exchanges are realized is modeled in such a large system by assuming that each actor tries to realize his or her best potential exchanges. In order to model this exchange process, four simplifying assumptions are made. The first assumption specifies the utility functions of actors on the outcomes. The second is related to the information actors possess during collective decision making. The third excludes strategic behavior, and the fourth is related to the exchange equilibrium the actors aim at. The assumptions are:

ASSUMPTION 1: SPECIFICATION OF UTILITY FUNCTIONS. *On each issue a, actors have single-peaked preference functions. The expected utility of actor i on some issue a is a function of the salience of the issue for the actor,* s_{ia}, *and the distance between the outcome of decision a and the policy position of actor i on decision a. Denoting the expected outcome of decision a as* O_a, *the expected utility for actor i on decision a is given by the following linear function:*[4]

$$EU^iO_a = -s_{ia}\,|O_a - x_{ia}|. \tag{1}$$

The total expected utility for actor i over all M issues is assumed to be the sum of his or her utilities over all issues:

$$EU^iO = \Sigma_a\, U^iO_a. \tag{2}$$

often used to denote the signed interest x. In our model, however, we need an extra symbol because the voting and policy positions of actors are unrelated to their interests in events. For that reason x^* and x are used to denote respectively the policy and voting position of an actor and the symbol s to denote his or her salience or interest.

4. Decisions can have very different ranges. For that reason, all decisions are normalized between 0 and 1 by dividing the policy and voting positions of actors through the range. The expected utility function is therefore defined on the normalized decision. Note that, except for the risk factor, this function is similar to the expected utility function of Bueno de Mesquita in chapter 4.

Assumption 2: Full Information. *Policy and voting positions, voting powers, and the saliences of all actors are assumed to be common knowledge. On the basis of these elements and the decision rules, all actors are able to compute the expected outcomes of decisions when no exchanges of voting positions take place.*

Assumption 3: No Strategic Behavior. *Strategic behavior of actors is not allowed. This has two implications:*

1. *Exchanges of voting positions on two issues a and b between two actors i and j are restricted to actors with policy positions on opposite sides of the expected outcomes of both issues:* $(x^*_{iu} - O_u)(x^*_{ju} - O_u) < 0$ *for* $u = a,b$.
2. *After the exchange, actors take voting positions in the interval* $[x^*_{iu}, x^*_{ju}]$ *for* $u = a,b$.

Assumption 4: Symmetry Equilibrium after Exchange. *When an actor exchanges voting positions with another actor, she or he accepts no smaller gain of expected utility than the other actor.*

Assumption 1 specifies the same loss function as that in the Bueno de Mesquita model if we disregard the risk-taking component. The risk-taking component makes it possible to model all kinds of strategic behavior. Its elimination from our loss function is justified because we purposely aim to exclude strategic behavior in our present, first elaboration of the model in order to investigate the effects of mutually beneficial exchanges between actors on pairs of decisions. Therefore, assumption 3 restricts the exchanges to those between two actors on opposite sides of the expected outcomes. For such pairs of actors, exchange of voting positions on two decisions is the sole possibility for a joint increase of utility, whereas actors on the same side have possibilities to increase their utility on only one decision without having to give something away on another decision (e.g., by choosing more extreme voting positions that are advantageous for both actors). In combination with assumption 3, assumption 2 is not very restrictive because incomplete information is primarily relevant for effective strategic behavior that will not immediately be observed and compensated for by other actors.

The fourth assumption specifies the symmetric effects that the actors aim to achieve through the exchange. Although the assumption of equal expected utility gain for both actors seems intuitively appealing, two objections can be made.

The first objection is that the assumption seems to imply a comparison of utilities between actors which is problematic and not allowed. Each actor does not compare his expected utility gain with that of the other, but with his perception of the expected utility gain of the other actor. By assumption 2, however, this perception is equal to the actual expected utility gain by the second actor. Here, but also later, we see that our model maximizes expected utilities of actors and not utility itself. It implies, however, that any extension of the model to include strategic behavior requires the explicit reformulation of assumption 4 in terms of the perceptions of the actors and not in terms of their actual utilities.

The second objection is that this equilibrium is based on intuitive reasoning and is not derived from a theory of the underlying micro-process. If we solely had to deal with the micro-process, a better alternative would have been available, but that alternative gives too many complications at the next stage of our model, namely, the selection of the realized exchanges from the pool of potential exchanges in a system of *N* actors and *M* decisions. That alternative is the Raiffa–Kalai–Smorodinsky solution given in bargaining theory (Friedman 1990, pp. 218–23). Reformulated in the context of our own research problem, actors compare their utility of the expected outcome without exchange (the so-called status quo situation) with the ideal situation, that is, the expected outcome if the other actor is prepared to vote for his or her policy positions on both issues. From a number of desirable axioms, an equal percentage of utility gain — instead of equal utility gain — is derived as the equilibrium after exchange. This solution is proven to be Pareto optimal for both actors, and it does not involve intersubjective comparison of utilities. In future extensions of our model, we aim to investigate the possibility of incorporating this exchange rate in our model, but we do not see the possibility to do so now.

In actual practice, the four assumptions are, of course, unrealistic. How serious this is can be observed when we apply the model to actual decision making and use our model to predict outcomes of decisions. If the model results in wrong predictions, we can relax the assumptions to build more complicated models according to the method of decreasing abstraction (Bueno de Mesquita 1981; Lindenberg 1990). The simplifying assumptions are therefore not essential for the chosen approach.

Collective decision making quite often consists not only of taking decisions on a set of prior issues, but also of the inventive creation of new choices. This is accomplished by splitting issues into a number of choices so as to provide for optimal compromises across the divergent interests of actors (Riker 1986). By properly specifying the salience actors attach to underlying dimensions instead of on the issues as a whole, our exchange model is able to predict the generation of these new choices.

Voting Power and Outcomes in Multistage Procedures

In general, the outcome of an issue depends on the policy positions of the actors, the decision rules, and the weights of the actors. In this section, we present a definition of voting power that takes into account both the weights of the actors and the decision rules. In the previous section we dealt with single-peaked, unidimensional decisions. The voting power measure is based on the assumption that such decisions in the formal voting procedure are converted to pro and con votes and that actors vote for the alternative that gives them the highest expected utility. The proposed voting power measure can be aggregated over different phases of a decision-making procedure. As such, the measure can fully represent the constitutional arrangements of the collective decision-making process, even when these are based on a multistage procedure.[5] Such a definition should be independent of more informal ways to influence outcomes of decisions or of exchanges of voting positions among actors. An appropriate definition of voting power was given by Stokman and Van den Bos (1992) who develop their concept of voting power precisely to represent the institutional settings as a separate element in their two-stage model of the political process.[6] They define the voting power of an actor as the proportion of collective decisions that is consistent with the policy position of the actor over all possible combinations of policy positions of the actors who participate in the decision process. According to this definition, the voting power of actors varies from .5 for actors without voting power to 1 for a dictator. For computational reasons, we rescale the voting powers to the interval from 0 to 1, resulting in a voting power of 0 for all actors without voting power. Actors with positive voting power are denoted as public actors.

As an example, let us consider the West German Bundestag after the 1987 general election. The simple majority criterion is 249 votes. As the German parties are rather homogeneous, we may consider them as

5. Such an aggregate measure is particularly useful for a global analysis of the decision-making process. In specific analyses of subprocesses with given policy positions of actors, prediction of the outcomes of decisions on the actual number of votes of actors and the specific decision rule might be preferable.
6. Their definition of voting power relies on the concept of decisional power of Hoede and his collaborators (Hoede and Bakker 1982; Hoede and Meek 1983; Hoede and Redfern 1985). Hoede and Bakker (1982) define the *decisional power* of an actor as the proportion of collective decisions that is consistent with his or her inclination over all possible combinations of inclinations of the actors. It depends on the decision rule (simple majority, qualified majority, or unanimity), the weights of the actors, and the control relations among the actors by which certain inclinations are converted to other preferences because of the existing control relations. The voting power of an actor in the Stokman–Van den Bos model deviates from Hoede and Bakker by modeling the conversion of certain inclinations via existing control relations to other preferences separately in the first stage of their model. Their definition of *voting power* is equivalent to that of decisional power in Hoede and Bakker when no influence relations among actors are taken into account. It is comparable to the Shapley-Shubik power index (Shapley and Shubik 1954).

the public actors in the Bundestag. The parties are the Christian Democratic Union/Christian Socialist Union (CDU/CSU) with 223 seats, the Social Democratic Pary (SDP) with 186, the Free Democratic Pary (FDP) with 46 seats, and the Green Party with 42 seats. The voting powers of the four actors are .75 for the CDU/CSU and .25 for each of the three other parties. The equal voting power for the three other parties is due to the fact that CDU/CSU can get a majority with each of them, whereas the only majority excluding the CDU/CSU requires the cooperation of all three parties.

The voting powers of the actors can be specified in a matrix, V. If we have N actors and M collective decisions, the order of the matrix is $(N \times M)$ and its entries v_{ia} specify the voting power of actor i with respect to decision a.

Stokman and Van den Bos also give an extension of the definition of voting power to multistage decision-making procedures. When research relates to more complicated decisions, as when different executive and legislative or supervisory boards are involved in the formal process, a single dimension may be insufficient for a proper representation of the institutional arrangements. To handle such situations, the voting power measure has been extended to enable the specification of several dimensions.[7]

Two examples may clarify the wide variety of institutional arrangements that can be represented in this way. First, let us extend the example of the German Bundestag. A legislative decision requires both the consent of the German government and that of the Bundestag (disregarding the senate for simplicity). After the 1987 election, a German government was formed consisting of a coalition of CDU/CSU and FDP. We assume that any legislative measure requires the consent of both coalition partners in the government. We can represent this multistage decision-making process by specifying two dimensions. On the first dimension (representing the German government) we give the CDU/CSU ministers and the FDP ministers an equal weight of one vote and specify as the decision criterion the need for two votes. That is, unanimity is required in the government. The second dimension represents the Bundestag and its specification is the same as above. The voting powers of the public actors now become: .25 for the CDU/CSU and for the FDP ministers, .186 for the CDU/CSU Bundestag fraction, and .0625 for the three other parties.

A second example illustrates how institutional arrangements in corporatist systems can be represented. Typical of corporatist systems is the requirement that all social partners simultaneously agree, and that the majority of parliament also agrees. This system can be depicted by spec-

7. This extension is due to Tom Snijders of the University of Groningen, who is also the author of a Pascal program to compute the voting power measure. In the program, up to five dimensions can be specified. The program is available on request.

ification of one dimension in which the corporatist, social partners have positive weights with unanimity as its decision rule. A second criterion indicates that the political parties are weighted by their number of seats in parliament, with simple majority as the decision rule. When relevant, a third criterion might be specified for the government if its consent is necessary for implementation of the decision, as when the government can exercise a veto.

In this definition of the voting power of actors, their respective weights and the decision rules in a multistage decision-making procedure are already incorporated. When using this measure, it seems inappropriate to take these elements into account again in predicting the outcome of a decision based on the voting positions and voting powers of the actors. In the Stokman–Van den Bos model, the outcomes of collective decisions are, therefore, predicted by taking the average of the voting positions of the public actors, weighted according to their voting power. The voting positions of actors need not be the same as their policy positions due to several processes in which public actors can adapt their voting positions. In Stokman–Van den Bos such adaptations are due to informal influence processes in the first stage of their model. Here we elaborate on adaptations or fluctuations in voting positions that result from exchanges or deals among public actors in the second stage. Following the notation given in the previous section, the predicted outcome of a decision a, O_a, is given by

$$O_a = (\Sigma_i x_{ia} v_{ia})/(\Sigma_i v_{ia}). \qquad (3)$$

When we apply equation 3 to the original policy positions of the actors, we denote the predicted outcome as the *base model*. Under the base model specification, the formal decision rules are applied to the original policy positions of the public actors without taking into account differences of interests among the actors and any changes due to informal processes or exchanges of voting positions. The base model can be seen as a kind of realistic null model. We expect that our models will predict decisions better than the base model

When the decision rule requires unanimity among all public actors (as is the case with almost all EC council decisions considered in this book),. all actors have equal voting power. Nevertheless, it is well known that in the EC council the policy positions of large member states informally have a larger weight than those of smaller member states. It is also evident that differences in interests among the member states are taken into account. Although it is our opinion that such differences should be modeled by focusing on the processes by which policy positions are transformed to voting positions, Van den Bos (1991) modeled these differences more or less directly by taking the number of votes assigned to each member state in majority decisions as voting weights and by including weights for the saliences of actors in the outcome function.[8] He called this model the *compromise model* and assumed that the president of the council would formulate a compromise in which the policy positions of the member states were weighted according to their votes and their saliences. The following alternative outcome function can be defined, representing the compromise model of Van den Bos:

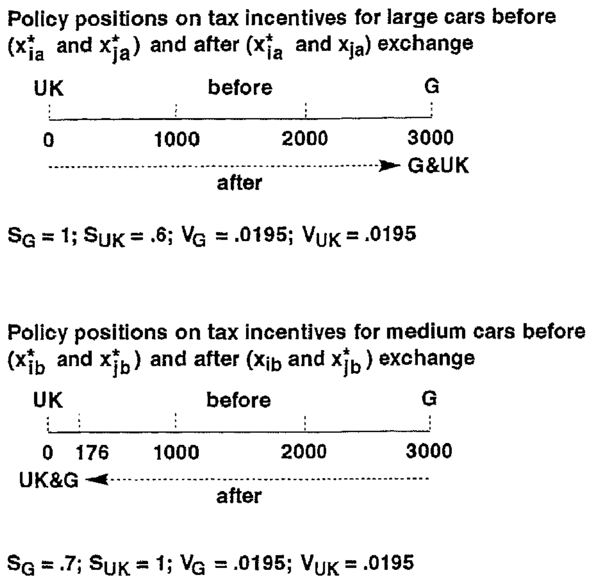
$$O'_a = (\Sigma_i x_{ia} v'_{ia} s_{ia})/(\Sigma_i v'_{ia} s_{ia}). \quad (4)$$

These two definitions of the predicted outcome as a weighted average of voting positions can be applied directly to decisions involving purely numerical issue positions, such as the amount of money to spend or the height of a public building. However, it is not possible to apply these relations to binary choices to accept a bill or not. In the Stokman–Van den Bos model, actors can take voting positions on the whole range of pro or con choices (which we designate as falling between +1 and −1), indicating their inclination to vote for or against a proposal. In the exchange model below, voting positions between −1 and +1 on such decisions are also allowed as inclinations to vote in favor or against. In these situations, the use of the weighted average of the voting positions, as given in equations 3 and 4, also seems appropriate for predicting outcomes. A pro outcome is then predicted when this average is positive, a con one when it is negative. The difference from zero may be interpreted as the probability that our prediction is correct.

Conditions For Exchange of Voting Positions Among Public Actors

To clarify the exchange of positions or logrolling in decision making, we will consider the simple case of two decisions — *a* and *b* — on which actors *i* and *j* have voting power. As an illustration, the policy positions (preferences), voting power, and saliences of Germany and the United Kingdom are given in figure 5.1 for two issues introduced in chapter 3: the amount of tax incentives in deutsche marks that EC countries are permitted to give for large and medium-sized cars that already fulfill strong exhaust emission standards. Germany and the United Kingdom have different policy positions on both issues. Germany favors high tax incentives, namely, DM 3,000. The United Kingdom wants to permit no tax incentives. The policy positions can be related to the different positions in which the respective car industries are situated. At the time of the decision (1987), the required technology was already available for the German car industry. They considered this technology to be necessary for German autos to remain competitive in markets outside Europe, whereas this was

8. In majority votes the member states have the following number of votes: France, Germany, Great Britain, and Italy, 10 each; Spain, 8; Belgium, Greece, Netherlands, and Portugal, 5 each; Denmark and Ireland, 3 each; and Luxembourg, 2.



5.1: An Exchange of Voting Positions: Outcome Function of the Base Model

not the case for the British car industry. The German car industry supported the position taken by its government, a position also dictated by the grave condition of German forests and the resulting public pressure to take measures to protect the environment. As the German automobile industry did not fear Japanese competition in the large-car market segment at that time, tax incentives (in combination with an early introduction date for strong emission standards) for that sector of the market had very high priority for Germany. The British automobile industry, however, primarily produces for the medium-car market segment, making the prohibition of tax incentives in combination with a late introduction date and low emission standards for that segment of the market very important. This is reflected in the saliences of the two countries for the two issues, as given in figure 5.1. Germany has a larger interest in tax incentives for large cars than for medium cars, whereas the opposite is the case for the United Kingdom.

To investigate the necessary conditions under which an exchange of voting positions between two actors is attractive, let us assume, for the sake of convenience, that actor i asks actor j to shift its voting position on issue a, whereas actor j asks actor i to do that in return on question b. We will denote a as the *demand issue* for actor i and the *supply issue* for actor j. Consequently, decision b is the demand issue for actor j and the supply issue for actor i. We formulate the following necessary conditions under which an exchange of voting positions between two actors is attractive to them:

THEOREM 1. *For two actors i and j with policy positions on opposite sides of the expected outcomes of two issues a and b, an exchange of voting positions on decisions a and b increases their overall expected utility only if the three conditions below are simultaneously met (decision a being the demand issue for actor i and decision b for actor j):*

1. $\Delta O_{ja}, \Delta O_{ib} > 0$, *where* ΔO_{ku} *denotes the change of outcome on decision u (u = a,b) brought about by a change of voting position of actor k (k = i,j) given no change in the voting positions of other actors so that for both actors, a change of voting position on the supply issue should result in a positive change in the expected outcome.*
2. $s_{ia}, s_{jb} > 0$, *so that both actors attach positive salience to the demand issue.*
3. $s_{ja} = 0$ *or*

$$\frac{s_{ib}}{s_{ia}} < \frac{s_{jb}}{s_{ja}} \qquad (\text{If } s_{ja} > 0),$$

so that either the salience on the supply issue for actor j is zero or else the ratio between actor i's saliences on his or her supply and demand issue is smaller than the ratio between actor j's saliences on his or her demand and supply issue.

Proof. Let us denote a shift by actor j on decision a as Δ_{ja}, where $x_{ja} = x^*_{ja} + \Delta x_{ja}$. Then a shift of Δx_{ja} in the direction of the policy position of actor i results in an increase in expected utility for actor i of EU^{i+} ΔO_{ja} $= \Delta O_{ja}s_{ia}$. The resulting decrease of expected utility for actor j is EU^{j-} $\Delta O_{ja} = \Delta O_{ja}s_{ja}$. To compensate for actor j's loss, actor i shifts Δx_{ib} on decision b in the direction of the policy position favored by actor j. This increases the expected utility of actor j by EU^{j+} $\Delta O_{ib} = \Delta O_{ib}s_{jb}$, and decreases the expected utility for actor i by EU^{i-} $\Delta O_{ib} = \Delta O_{ib}s_{ib}$. Exchange is attractive only if actor i and j can improve their expected utility simultaneously. This is true if both $EU^{i+} > EU^{i-}$ and $EU^{j+} > EU^{j-}$. In other words, both $\Delta O_{ja}s_{ia} - \Delta O_{ib}s_{ib} > 0$ and $\Delta O_{ib}s_{jb} - \Delta O_{ja}s_{ja} > 0$. This implies, first of all, that $\Delta O_{ja},\Delta O_{ib},s_{jb},s_{ia} > 0$, the first two conditions of the theorem. Moreover, in combination with the first two conditions, actor i gets an expected utility gain only if

$$\Delta O_{ja}/\Delta O_{ib} > s_{ib}/s_{ia}.$$

Similarly, actor j gets an expected utility gain in combination with the first two conditions only if $s_{ja} = 0$ or

$\Delta O_{ja}/\Delta O_{ib} < s_{jb}/s_{ia}$ for $s_{ja} > 0$.

Combined they give the third condition in the theorem. QED

When two issues meet the three conditions for two actors, the conditions uniquely determine which issue is the demand issue for one actor and which is the demand issue for the other actor. If more actors and decisions are considered, however, a given issue can simultaneously be a demand issue in one potential exchange and a supply decision in another potential exchange for the same actor.

Condition 1 from theorem 1 can easily be specified for the outcome functions in equations 3 and 4. The outcome function in equation 3 holds that the voting power on the supply issue should be positive for both actors. The outcome function in equation 4 holds that the product of voting power and salience on the supply issue should be positive for both actors. These results are given in the following two corollaries.

COROLLARY 1.1. *If the outcome function given in equation 3 is used, condition 1 in theorem 1 can be specified as* v_{ja}, $v_{ib} > 0$ *if a is the demand issue for actor i, and as* v_{ia}, $v_{jb} > 0$ *if a is the demand issue for actor j.*

Proof. Assume that a is the demand decision of actor i and that actor j is the only actor shifting its position on a. According to equation 3 with O_a^* denoting the outcome on a after the exchange

$$O_a^* = \frac{\Sigma_{k \neq j} x_{ka}^* v_{ka} + (x_{ja}^* + \Delta x_{ja}) v_{ja}}{\Sigma_k v_{ka}}$$

$$= \frac{\Sigma_k x_{ka}^* v_{ka}}{\Sigma_k v_{ka}} + \frac{\Delta x_{ja} v_{ja}}{\Sigma_k v_{ka}}$$

$$= O_a + \frac{\Delta x_{ja} v_{ja}}{\Sigma_m v_{ma}} . \qquad (5)$$

Since by assumption, $\Delta x_{ja} > 0$, $|\Delta O_{ja}| > 0 \Leftrightarrow v_{ja} > 0$. In an analogous way it can be proved that $v_{ib} > 0$. QED

COROLLARY 1.2. *If the outcome function given in equation 4 is used, condition 3 in theorem 1 can be specified as* $(v'_{ja} s_{ja})$, $(v'_{ib} s_{ib}) > 0$ *if a is the demand issue for actor i, and as* $(v'_{ia} s_{ia})$, $(v'_{jb} s_{jb}) > 0$ *if a is the demand decision for actor j.*

The proof of corollary 1.2 is analogous to the proof of corollary 1.1.

In figure 5.1, Germany and the United Kingdom are at opposite ends of the policy spectrum on both issues, with the expected outcome at an intermediate position. Moreover, if the large car issue is taken as the demand issue for Germany and the medium-sized car issue as the demand issue for United Kingdom, we observe that the three conditions of theorem 1 are met:

- The two actors have positive voting power on the supply issue (in fact they have voting power on both).
- The two actors have positive saliences on their demand issues.
- The ratio between the supply issue and demand issue for Germany is smaller than the ratio between the demand and supply issue for the United Kingdom (for Germany the ratio is .7; for the United Kingdom 1.7).

The next problem is to specify the state of equilibrium after a position exchange. This requires the specification of the exchange rate between the voting positions and the determination of the total size of the exchange.

THEOREM 2. *If for two decisions a and b and two actors i and j the conditions of theorem 1 are met and if a is the demand issue for actor i, then the exchange rate between the shifts of outcomes on a and b is*

$$\Delta O_{ja} = \frac{(s_{ib} + s_{jb})}{(s_{ia} + s_{ja})}\Delta O_{ib}. \tag{6}$$

Proof. By assumption, a is the demand decision for actor i. A shift of Δx_{ja} in the direction of the policy position of actor i results in an increase in expected utility for actor i of $EU^{i+}\ \Delta O_{ja} = \Delta O_{ja}s_{ia}$. The resulting decrease in expected utility for actor j is $EU^{j-}\ \Delta O_{ja} = \Delta O_{ja}s_{ja}$. To compensate for actor j's loss, actor i shifts Δx_{ib} on issue b in the direction of the policy position favored by actor j. This increases the expected utility of actor j by $EU^{j+}\ \Delta O_{ib} = \Delta O_{ib}s_{jb}$ and decreases the expected utility for actor i by $EU^{i-}\ \Delta O_{ib} = \Delta O_{ib}s_{ib}$.

For the two actors the net improvement in expected utility will be equal (assumption 4) if

$$EU^{i+}\Delta O_{ja} - EU^{i-}\Delta O_{ib} = EU^{j+}\Delta O_{ib} - EU^{j-}\Delta O_{ja}$$

or

$$\Delta O_{ja}s_{ia} - \Delta O_{ib}s_{ib} = \Delta O_{ib}s_{jb} - \Delta O_{ja}s_{ja}$$

$$\Delta O_{ja}(s_{ia} + s_{ja}) = \Delta O_{ib}(s_{jb} + s_{ib})$$

$$\Delta O_{ja} = \frac{(s_{ib} + s_{jb})}{(s_{ia} + s_{ja})}\Delta O_{ib}. \quad \text{QED}$$

Theorem 2 expresses the exchange rate in terms of the outcomes on the issues. The necessary shifts of voting positions by the two actors depend on the decision rules and the weights of the actors. Corollary 2.1 gives the exchange rates between the shifts in voting positions of the actors if the outcome function of equation 3 is valid, whereas corollary 2.2 does the same if the outcome function of equation 4 is applied.

COROLLARY 2.1. *If the outcome function of equation 3 is used and if issue a is the demand decision for actor i, then the exchange rate for the shifts in voting positions of actors i and j is*

$$\Delta x_{ja} = \frac{(s_{ib} + s_{jb})v_{ib}\Sigma_k v_{ka}}{(s_{ia} + s_{ja})v_{ja}\Sigma_k v_{kb}}\Delta x_{ib}. \tag{7}$$

Proof. The relation between the shifts in outcomes of the decisions and the shifts in voting positions is now given by

$$\Delta O_{ja} = \frac{\Delta x_{ja} v_{ja}}{\Sigma_k v_{ka}}$$

and

$$\Delta O_{ib} = \frac{\Delta x_{ib} v_{ib}}{\Sigma_k v_{kb}}.$$

Substitution gives equation 7. QED

COROLLARY 2.2. *If the outcome function of equation 4 is used and if a is the demand decision for actor i, the exchange rate for the shifts in voting positions of actors i and j is*

$$\Delta x_{ja} = \frac{(s_{ib} + s_{jb})v'_{ib}s_{ib}\Sigma_k v'_{ka}s_{ka}}{(s_{ia} + s_{ja})v'_{ja}s_{ja}\Sigma_k v'_{kb}s_{kb}}\Delta x_{ib}. \tag{8}$$

Proof. From equation 4, it follows that

$$\Delta O_{ja} = \frac{\Delta x_{ja} v'_{ja} s_{ja}}{\Sigma_k v'_{ka} s_{ka}}$$

and

$$\Delta O_{ib} = \frac{\Delta x_{ib} v'_{ib} s_{ib}}{\Sigma_k v'_{kb} s_{kb}}$$

Substitution gives equation 8. QED

The corollaries show an important aspect of the exchange process, namely, that the exchange rates between actors is dependent on the decision rule and the weights of the actors as reflected in the outcome function. In other words, different decision rules and weights for the actors can lead to different exchanges! Under the outcome function of the base model (equation 3), the exchange rate between the shifts in voting positions of the United Kingdom on the large car issue and Germany on the medium-sized car issue in the example of figure 5.1 is $(1 + .7)/(1 + .6) = 1.063$ because the voting power of the two actors and the sum of the voting powers of all actors is assumed to be the same for the two decisions.[9]

9. The latter is not always the case, as Hoede et al. demonstrated. The voting power of

When the outcome function of the compromise model (equation 4) is used (or is perceived to be used), the summation terms in equation 4 are not equal for issues *a* and *b*. Taking the relevant salience data from table 3.3 and the voting weights as given in note 8, the United Kingdom and Germany would apply another exchange rate,[10] namely, $((1 + .7) \cdot 7 \cdot 31.4)/((1 + .6) \cdot 6 \cdot 38.4) = 1.014$.

What remains to be solved is determining the new voting positions after the exchange takes place. Since we assume that the utility for alternative voting positions decreases on either side of an actor's stated policy position, actors are not interested in shifting the outcome across and beyond their policy position. Such an outcome might require a shift in the voting position of the other actor far beyond one's own policy position. However, this strategic behavior is excluded by assumption 3 where the shifts are confined to the interval between the two policy positions. The total amount of exchange is therefore determined by whichever of the two following conditions is met first:

$$\Delta x_{ja} \leq |x^*_{ia} - x^*_{ja}|$$

and (9)

$$\Delta x_{ib} \leq |x^*_{ib} - x^*_{jb}|.$$

For the example found in figure 5.1, the resulting voting positions of the United Kingdom and Germany on the two issues are given in the last two parts of the figure under the outcome function of the base model (equation 3). Both actors realize an expected utility gain of .043 through this exchange. As decisions on these issues were taken before the entrance of Spain and Portugal into the EC, and Belgium and Greece are excluded from the computations because of missing policy positions, each of the remaining countries contribute .125 to the outcome. Normalizing the decision on the interval between 0 and 1, the net expected utility gain for Germany is $(1 \cdot 1 \cdot .125) - (.7 \cdot .941 \cdot .125) = .043$, and that for the United Kingdom $(1 \cdot .941 \cdot .125) - (.6 \cdot 1 \cdot .125) = .043$.

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a system is maximal when all actors have equal weights and decisions are made by simple majority. Hoede et al. also derive several interesting results regarding total voting power in representational systems. For example, the present system in the General Assembly of the United Nations (one vote for each member state) gives maximal voting power only if all member states are ruled by dictators. At the time of the indirect elections for the European Parliament, they derived that the maximal voting power for the European Parliament would be obtained when the ratios between the numbers of national representatives are equal to the square root of the sizes of the national electorates.

10. Because of missing values for Belgium and Greece they are excluded from the computations.

A Dynamic Exchange Model

The most important characteristic of social processes in general, and collective decision-making processes in particular, is the fact that the outcomes of "macro" processes are not the result of a central (planning) authority in the policy domain. Rather, outcomes are the intended or unintended consequences of the simultaneous choices of decision makers. Actors try to realize their goals by choosing between the behavioral alternatives that are available to them under certain restrictions. This is the core principle in the structural individualistic approach (Boudon and Bourricaud 1982; Coleman 1986; Lindenberg 1985; Wippler 1978). The rationality of the decision makers that is implied in this principle, however, is seriously hampered by the fact that the actors are presumed to act simultaneously. This implies that their rationally chosen alternatives might appear to be suboptimal because they did not anticipate the actions of other actors. This limitation is reinforced because the actors, in contrast to central (planning) authorities, have limited information about the system and the intended actions of other decision makers. This makes it inevitable for actors in certain situations to define instrumental goals that are only roughly related to the ultimate instrumental goals in the system and to make ex ante assumptions that turn out to be unrealistic ex post (e.g., that other actors behave in a certain way). Actors, however, evaluate the ultimate success of these derived strategies and assumptions and have the possibility to adapt them in case of frequent failure. In other words, actors have the ability of adaptive learning from experience. This latter feature can be used to facilitate the design of an applied model by starting with a very simple model and then, in a stepwise process, adding assumptions when the simple model fails. This method of model construction is known as the method of decreasing abstraction (Lindenberg 1992).

For our exchange problem we need to generalize our given solutions for two actors and two decisions to a system of N actors and M decisions. In classical model building, this step would consist of formulating equations on the resulting equilibrium at the macro level, which is quite complicated and often not solvable. Since we are interested in designing a model that can be applied in practice as well as in principle, we eschew that approach. The recently available computer methodology of object-oriented modeling makes it possible to arrive at a direct representation of such a physical world of parallel operating actors (Goldberg and Robson 1983). In object-oriented models, these actors are represented by objects. Like actors in the physical world, they have an internal structure that enables them to reason and to communicate with other objects. As in the physical world, the reasoning of and communications between objects in object-oriented models may take place simultaneously, may result in a diversity of actions by different objects depending on the restrictions under which they operate, and may be adapted on the basis of past experiences (Lehrmann Madsen and Moller-Pedersen 1988). As such, object-oriented modeling incorporates the ideas of parallel distributed processing, whereas the representation of actors as adaptive learning objects gives these objects characteristics similar to self-organizing systems, such as are known from neural networks (see, e.g., Rumelhart and McClelland 1987).[11] The characteristics of social systems that are emphasized in the structural individualistic approach (parallel, operating under different restrictions, self-learning, and actors reacting to each other on the basis of which social phenomena develop) have their direct equivalences in object-oriented models. In other words, the principles of the structural individualistic approach and those of object-oriented modeling are so strikingly similar that object-oriented programming is seen as the appropriate means for an adequate representation of collective decision-making processes in general and exchange processes in particular. Object-oriented modeling permits the development of an applied model, one that can serve as a practical tool as well as an analytic construct.

The generalization of the exchange problem to a system of N actors and M decisions in the object-oriented model is solved by letting the actors negotiate with each other on the basis of the limited information that is available to them. This process is facilitated by the fact that the results of the exchange process can be represented as a network between actors. Object-oriented modeling facilitates the re-use of classes of objects. The necessary classes of objects for a network (graph, points, and edges) were previously developed,[12] so that these classes can be used directly to represent the result of the exchange process. Moreover, in a system of M issues not all prospective decisions may be suitable for exchanges of voting positions within a policy domain. For example, exchanges of voting positions on two issues might be restricted to decisions that are taken at a certain time interval, one after the other, or to decisions over which the contents are somehow related. The complexity of the system of N actors and M decisions can therefore sometimes be reduced by defining a network among decisions that delimits the pairs of issues that are suitable for

11. The same guiding principles are used in a Ph.D. project on "Structure and dynamics of friendship networks in heterogeneous groups" by Evelien Zeggelink (1993), supervised by Frans N. Stokman, Cees Hoede, and Tom Snijders (ICS, Groningen).

12. The network classes and the main elements of the computer application for the exchange process were developed by Reinier Van Oosten in Smalltalk-80. Smalltalk is the programming language that made the object-oriented paradigm popular. It is object-oriented to the extreme: everything in the system is an object. Smalltalk is not just a language, but also a very powerful programming environment for the language. Its powerful editing facilities, its powerful debugger, and its incremental compilation make Smalltalk a perfect fit for explorative and experimental research on social networks.

logrolling or exchanges. Again, the basic network classes can be used directly for these purposes.

We assume that actors are primarily interested in exchanges on pairs of issues on which they believe they stand to gain a lot. Because of assumption 4, the gains for the two actors are the same, which implies that an ordering of the exchanges in the system as a whole is at the same time an ordering of the exchanges for each individual actor. This would not be the case if the exchange rate had been determined on the basis of an equal percentage of utility gain for the two actors, the solution of the Raiffa-Kalai-Smorodinsky bargaining approach. It is for this reason that their solution creates problems in this stage of the modeling process and why at present the equal gain assumption is maintained.

The net gain on an exchange is highest if the gain on the demand decision is large and the loss on the supply decision is small. The first thing actors do in the model is to make a *demand list:* the actor orders the issues in terms of the maximal gain she or he can get. The more extreme the policy position of an actor on an issue and the higher its salience, the more an actor can gain by persuading other actors to shift their voting positions.

The exchange algorithm can now be summarized as follows:

1. Create a new network of potential exchange relations with the actors as its points.
2. Between any two issues on which actors are allowed to exchange voting positions, determine for each pair of actors whether they have policy positions on opposite sides of the expected outcomes of the issues and whether the three conditions of theorem 1 are fulfilled. For any pair of issues on which the pair of actors can exchange voting positions, create an edge (line) between the two actors in the network of possible exchange relations.
3. For each edge, determine the demand and supply decisions for actors i and j. Suppose (as we did above), the demand decision is issue a for i and b for j (that is, actor i is willing to move in the direction of actor j on issue b, and actor j is willing to do so on a).
4. For each edge, determine the maximal potential improvement of expected utility for the actors (which is the same for the two actors by theorem 2).
5. Order the edges in the network of potential exchange relations by the size of the maximal potential improvement in expected utility.
6. Define a new network of realized exchange relations with the actors as its points.
7. The edge with the highest maximal potential improvement of ex-

pected utility is the first realized exchange. It is moved to the network of realized exchange relations and deleted from the network of potential exchange relations. When several edges have the same value, one is selected at random.

8. Delete all other edges in the network of potential exchange relations for which decision *b* was the supply issue for actor *i*. Do the same for actor *j* regarding its supply decision *a*.
9. Continue steps 7 and 8 until no edges are left in the network of potential exchange relations.
10. Determine the voting positions of all actors on all decisions after exchange and compute the policy outcomes on the issues.
11. If random decisions have been made in step 7, repeat steps 5 to 9 several times to determine the means and standard deviations of the voting positions of the actors and the outcomes on the issues.

If no random choices between potential exchanges are made in step 7, the exchange algorithm results in a unique set of exchanges and a unique prediction of the resolution of the issues. The expected utility gain for each actor over all exchanges can be determined by cumulating his or her expected utility gain for each realized exchange. The total expected utility gain for the whole set of actors is then obtained by summing these expected gains over all actors. It should be realized, however, that these quantities do not represent the realized utility gains. If two actors realize an exchange, its effects on the resolution of the two issues result in utility gains and losses for all actors. Moreover, other exchanges among other actors may well nullify or enlarge the contemplated changes in outcomes and the expected utility gains for the exchange partners. Therefore, the realized utility gain for each actor and for the set of actors as a whole is another criterion to be taken into account in our judgment on the results of the exchange process. For example, as one of the analyses in chapter 7 shows, the final outcome may well result in an overall loss of utility and may, therefore, not be Pareto optimal for the set of all actors.

If random choices are made in step 7, the set of realized exchanges is not unique. Of course, different sets of exchanges may well result in the same predicted issue outcomes so that a unique prediction of the outcomes may result even in this case. In other situations, however, the predicted outcomes differ over different sets of realized exchanges. Then, the means of the predicted outcomes over one hundred simulations are taken as our predictions. Special attention will be given, however, to solutions that result in maximal expected utility gains and maximal realized utility gains for the set of all actors. The analyses in chapter 7 show that these two criteria vary quite independently from one another. We prefer the mean solution to one of these two because it is hard to defend a notion that suggests that actors optimize either the expected or the realized utility for the system as a whole instead of their own utility. In situations of uncertainty, the mean reduces the uncertainty more than any other statistic.

The exchange algorithm is extremely simple. First of all, actors do not anticipate the effects of other exchanges. In a more complicated model, we might give actors the ability to investigate whether certain seemingly less profitable exchanges should be given priority because these exchanges will block exchanges by the other actor in one or another way. Second, all exchanges and their expected utilities are related to the original expected outcomes of the decisions. An alternative would be to give each actor the opportunity to do his or her best exchange and to recalculate the expected utilities of the remaining potential exchanges in terms of the adapted expected outcomes before a second exchange is allowed.

The exchange model of voting positions developed in this chapter deviates fundamentally from existing exchange models. It takes the core aspects of collective decision as points of departure: the exchange of voting positions rather than the exchange of control; actors with single-peaked preference functions rather than linear increasing preference functions; multistage decision procedures rather than single-stage decisions. This model yields a number of relevant results:

1. a well-founded measure for the voting power of actors in multistage decision-making procedures;
2. the conditions under which exchange of voting positions is attractive for actors;
3. the prediction of exchange rates that actors will use; and
4. a direct representation of the negotiating process by the application of object-oriented modeling techniques.

In the models presented in this book, the exchange rates between voting positions are determined at the level of the pairs of actors; no prices at the system level emerge. This might be seen as an important disadvantage when compared with other exchange models of collective decision making. However, the absence of physical currency and prices at the system (market) level is seen as a fundamental aspect of exchanges in political systems by other authors as well (Parsons and Smelser 1956; Coleman 1970; Marsden 1983). According to Marsden, the absence of a physical currency and the inalienability of official decision-making authority (voting power)

> mean that trust in the operation of political exchange must be guaranteed in a manner distinct from that used in economic exchange. Bargains struck may involve one actor using resources in the interest of another at one time in

> exchange for a promised reciprocation by the other actor. These bargains are different from economic exchanges because the actor receiving political value does not surrender constitutional control over an equivalent amount of value at the time it is received. Instead, a promise is made concerning the disposition of equivalent value at a future time. But no mechanism such as money is available to insure that the promised future action will indeed be forthcoming. (Marsden 1983, p. 691)

From this perspective, the determination of the exchange rates at the level of the pairs of actors rather than at the system level can be seen as appropriate and advantageous to our model, facilitating its application in a variety of contexts. For example, the model can be applied to informal influence processes within policy networks. The exchange described above was confined to actors with voting power. In the phase before the final decision, however, opinion formation takes place in interactions among public and private actors. This results in indirect voting power for influential private actors, a process that was explicitly modeled by Laumann and Knoke (1987), as well as Stokman and Van den Bos (1992). Exchanges of positions among private actors with complementary interests may lead to a joint approach toward public actors. With the help of an extension of the model presented here, it is possible to explain this important — though often neglected — aspect of collective decision-making processes. This can be made more realistic by using information on informal networks. In fact, one can restrict the model to actors who are connected by the informal network or by other network ties. This possibility has been explicitly used by Marsden (1983).

In addition, the model can be applied to transaction costs (Williamson 1991). Transaction costs in collective decision-making processes can be assumed to depend on several factors. First, characteristics of the pair of decisions might be influential for transaction costs. If two decisions are to be taken within a large time interval, exchange of voting positions involves extra risks for the actor for whom the second decision is the demand decision. These risks depend also on characteristics of the two actors and on the relation among them. As our exchange model is based on a combination of networks among decisions and among actors, all relevant types of transaction costs can be taken into account and related to the exchange rates.